

ASYMPTOTIC BEHAVIOR OF NUCLEON ELECTROMAGNETIC FORM FACTORS IN SPACE- AND TIME-LIKE REGIONS

Egle Tomasi-Gustafsson⁽¹⁾ and Michail P. Rekalo⁽²⁾

(1) *DAPNIA/SPhN, CEA/Saclay, 91191 Gif-sur-Yvette Cedex, France*

(2) *Middle East Technical University, Physics Department, Ankara 06531,
Turkey*

and

National Science Center KFTI, 310108 Kharkov, Ukraine

Abstract

We compare the existing data about electromagnetic form factors of the proton in the time-like region of momentum transfer (up to $q^2 \simeq 14$ (GeV/c)²), with the corresponding data in the space-like region. From the constraints given by the Phragmén-Lindelöf theorem, it turns out that the asymptotic regime can not be reached simultaneously for both form factors (electric and magnetic) in the considered region of momentum transfer.

The angular dependence for the annihilation processes, such as $\bar{p} + p \leftrightarrow e^+ + e^-$, is sensitive to the asymptotic properties of form factors and to the two-photon physics in the time-like region.

Recent experimental data on nucleon electromagnetic form factors (FF) in time-like (TL) [1] and space-like (SL) [2] regions of momentum transfer square q^2 , and new theoretical developments [3] show the necessity of a global description of form factors in the full region of q^2 . The question of "where the asymptotic behavior for form factors is reached" is often discussed in literature, however different expectations and predictions are given by different models (for a recent review, see [3]). Our aim is to discuss the experimental data about proton electromagnetic structure in SL and TL regions, to estimate (independently from current models) where the asymptotic region is, and, finally, to suggest some observables to be measured in order to test these predictions.

For the description of the nucleon electromagnetic structure in the full region of momentum transfer square (space-like and time-like), two form factors are defined, electric, G_E , and magnetic, G_M , which are related to the Dirac (F_1) and Pauli (F_2) FFs by:

$$G_E = F + \tau F_2, \quad G_M = F_1 + F_2, \quad \tau = \frac{s}{4m^2},$$

where m is the nucleon mass and s is the square of the total energy in $e^+ + e^-$ ($p + \bar{p}$) collisions, i.e. the time-like equivalent of the four-momentum transfer square q^2 for $e + p$ -scattering.

In the QCD-approach, two aspects are related to the asymptotic region: 1) the q^2 -dependence of the electromagnetic hadronic form factors, in accordance with the quark counting rule [4, 5] and 2) the conservation of hadron helicity [6]. The existing experimental data about electromagnetic form factors of pion, nucleon and deuteron confirm the quark counting behavior: $F_A(q^2) \simeq 1/(q^2)^{n_A-1}$ where n_A is the number of elementary constituents in the hadron A . On the other hand the hypothesis of helicity conservation, which constrains in particular polarization observables, does not work successfully, for the deuteron form factors [7] and especially for the electromagnetic form factors of $N \rightarrow \Delta$ transitions [8], where the electric quadrupole (transversal and longitudinal) form factors, measured up to $q^2 = 4 \text{ (GeV/c)}^2$, are very small, in evident contradiction with helicity conservation. This situation is confirmed also by the study of different electromagnetic processes: pion photoproduction, $\gamma + N \rightarrow N + \pi$ [9], nucleon Compton scattering, $\gamma + p \rightarrow \gamma + p$ [10], deuteron photodisintegration, $\gamma + d \rightarrow n + p$ [11] and coherent π^0 -photoproduction, $\gamma + d \rightarrow d + \pi^0$ [12]. Note that the scaling behavior of the differential cross section is especially evident in elastic pp -scattering [13], where the data are consistent with the scaling law predictions over a wide range of angles and energies.

The comparison of the nucleon electromagnetic FF in SL and TL regions opens the way to a general and model independent discussion of asymptotic properties. Form factors are analytical functions of q^2 , being real functions in the SL region (due to the hermiticity of the electromagnetic Hamiltonian) and complex functions in the TL region. The Phragmén-Lindelöf theorem [14] gives a rigorous prescription for the asymptotic behavior of analytical functions: $\lim_{q^2 \rightarrow -\infty} F^{(SL)}(q^2) = \lim_{q^2 \rightarrow \infty} F^{(TL)}(q^2)$. This means that, asymptotically, the FFs, have the following constraints: 1) the time-like phase vanishes

and 2) the real part of the FFs, $\mathcal{R}eF^{(TL)}(s)$, coincides with the corresponding value $F^{(SL)}(q^2)$.

The existing experimental data about the electromagnetic FFs of charged pion or proton in the time-like region do not allow a similar complete test of the Phragmen-Lindelöf theorem, especially concerning the vanishing phase. Even in the simplest case of the pion form factor, $F_\pi(s)$, its phase (which is the physical quantity fixed by the unitarity condition) can not be derived from the cross section of the $e^+ + e^- \rightarrow \pi^+ + \pi^-$ process, as it depends on $|F_\pi(s)|^2$, only. This is also true for the proton FFs. However $T - odd$ polarization phenomena in $e^+ + e^- \leftrightarrow \bar{p} + p$ are sensitive to the relative phase δ of the electric and magnetic FFs. The simplest polarization observables, which are characterized by the product $\mathcal{I}mG_E G_M^* \simeq \sin \delta$ are the asymmetries in the reactions $\vec{e}^+ + e^- \rightarrow \bar{p} + p$ and $\bar{p} + \vec{p} \rightarrow e^+ + e^-$. However no polarization observables in these reactions allow to extract the absolute value of the phase. Only the study of more complicated reactions such as $\pi^- + p \rightarrow n + e^+ + e^-$ [15] or $\bar{p} + p \rightarrow \pi^0 + e^+ + e^-$ [16] allows, in principle:

- to determine the nucleon FFs in the unphysical region of TL momentum transfer, for $4m_e^2 \leq s \leq 4m^2$, where m_e is the leptonic mass;
- to determine the relative phase of pion and nucleon form factors.

In this work we focus on $e^- + p$ -elastic scattering and on the processes $e^+ + e^- \leftrightarrow \bar{p} + p$. We suggest here a new analysis of FFs in TL region and discuss experimental methods to check the different assumptions.

The recent measurement, in TL region, of the experimental cross section in the annihilation process $\bar{p} + p \rightarrow e^+ + e^-$ extends up to $s \simeq 14$ (GeV/c)². The cross section can be expressed as a function of FFs according to the following formula [17]:

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{8m^2\tau\sqrt{\tau(\tau-1)}} \left[\tau|G_M|^2(1 + \cos^2\theta) + |G_E|^2\sin^2\theta \right], \quad (1)$$

where θ is the angle between the electron and the antiproton in the center of mass frame. The angular dependence of the differential cross section can be written as:

$$\frac{d\sigma}{d(\cos\theta)} = \sigma_0 \left[1 + \mathcal{A}\cos^2\theta \right], \quad (2)$$

where σ_0 is the value of the differential cross section at $\theta = \pi/2$ and the

asymmetry \mathcal{A} , can be expressed as a function of the FFs:

$$\mathcal{A} = \frac{\tau|G_M|^2 - |G_E|^2}{\tau|G_M|^2 + |G_E|^2} \quad (3)$$

The Rosenbluth separation of $|G_E|^2$ and $|G_M|^2$ in TL region, has not been realized yet.

In order to extract the form factors, due to the poor statistics, it is necessary to integrate the differential cross section over a wide angular range. One assumes that the G_E -contribution plays a minor role in the cross section at large s and the experimental results are usually given in terms of $|G_M|$, under the hypothesis that $G_E = 0$ (case 1) or $|G_E| = |G_M|$ (case 2). The second hypothesis is strictly valid at threshold only, but there is no theoretical argument which justifies its validity at any other momentum transfer, where $s \neq 4m^2$. The first hypothesis is arbitrary.

The $|G_M|^2$ values depend, in principle, on the kinematics where the measurement was performed and the angular range of integration, however it turns out that these two assumptions for G_E lead to comparable values for $|G_M|$.

In the SL region the situation is different. The cross section for the elastic scattering of electron on protons is sufficiently large to allow the measurements of angular distribution and/or of polarization observables. The existing data on G_M show a dipole behavior up to the highest measured value, $q^2 \simeq 31 \text{ (GeV/c)}^2$ according to

$$G_M(q^2)/\mu_p = G_d, \text{ with } G_d = \frac{1}{\left[1 + \frac{q^2}{m_d^2}\right]^2}, \quad m_d^2 = 0.71 \text{ (GeV/c)}^2. \quad (4)$$

It should be noticed that the independent determination of both G_M and G_E FFs, from the unpolarized $e^- + p$ -cross section, has been done up to $q^2 = 8.7 \text{ (GeV/c)}^2$ [18], and the further extraction of G_M [19] assumes $G_E = G_M/\mu_p$. The behavior of G_E , deduced from polarization experiment $p(\vec{e}, e'\vec{p})$ differs from G_M/μ_p , with a deviation from G_d up to 50% at $q^2=3.5 \text{ (GeV/c)}^2$ [2]. This is the maximum momentum at which the new, precise data are available, which corresponds to values of s just under threshold of the reaction $\bar{p} + p \rightarrow e^+ + e^-$, when translated to TL region. The new data can still be fitted by a dipole function, also, but with a smaller value of $m_d^2 = 0.61 \text{ (GeV/c)}^2$.

The experimental situation is summarized in Fig.1, where the data are normalized to the function G_d . The values of G_M in the TL region, obtained under the assumption that $|G_E| = |G_M|$, are larger than the corresponding SL values. This has been considered as a proof of the non applicability of the Phragmen-Lindelöf theorem, or as an evidence that the asymptotic regime is not reached [20].

The magnetic form factor of the proton in the TL region (which is deduced from the hypothesis $G_E = 0$ or $G_E = G_M$), can be parametrized as: $G_M^{(TL)} = G_d \frac{a}{\left(1 + \frac{s}{m_{nd}^2}\right)^2}$, where a is a normalization parameter and $m_{nd}^2 = 3.6 \pm 0.9$ (GeV/c)² characterizes the deviation from the usual dipole s -dependence. The extrapolation to high q^2 based on this formula (Fig. 1, full line), indicates that the Phragmen-Lindelöf theorem will be satisfied by this FF, only for $s(q^2) \geq 20$ (GeV/c)².

The value of the mass parameter m_{nd}^2 is comparable for other electromagnetic form factors: the electric FF of the proton ($m^2 \simeq 5.3$ (GeV/c)²) (Fig. 1, dashed line) and the magnetic FF of the $N \rightarrow \Delta$ transition ($m^2 \simeq 6.1$ (GeV/c)²). This might be an indication that this parameter is related to the internal hadronic electromagnetic structure.

Let us consider now another procedure for the extraction of FF in the TL region assuming that at least one of the two proton electromagnetic FFs has reached the asymptotic regime. This looks as a reasonable hypothesis for G_M , which shows an early scaling behavior, in accordance with quark counting rules.

In this case the Phragmen-Lindelöf theorem constrains definitely $|G_M|$ in TL region to have the same value as in SL, and therefore from Eq. (1) we can deduce $|G_E|$, using the existing experimental data about $\bar{p} + p \leftrightarrow e^+ + e^-$ (case 3). A fourth possibility is taking for G_E in the TL region the values suggested from the SL region (i.e. assume that G_E is asymptotical in the considered region), and calculate $|G_M|$ (case 4).

We report, in Fig. 2, some of the recent data in TL region, reanalyzed following the possibilities suggested above. Fig. 2a shows the values of the form factors. For case 3, where G_M is taken according to Eq. 4 (G_M is asymptotical), the values of $|G_E|$ are plotted and they are larger than in cases 1 and 2. This seems to suggest that asymptotics are not reached for G_E , as the values in SL and TL gets more apart.

On the other hand, taking for $|G_E|$ the SL values (case 4), affects very little the values of G_M , due to the kinematical factor τ , which weights the magnetic contribution to the differential cross section.

Therefore, the existing data about $\bar{p} + p \rightarrow e^+ + e^-$ do not contradict the hypothesis that one form factor (electric or magnetic) could be asymptotic at relatively large momentum transfer ($s \simeq 6 \div 14$ (GeV/c)²), but in this case the other form factor is far from the asymptotic regime. Although affected by large statistical errors, the existing angular distributions in TL region [23, 24] do not contradict the possibility that the electric and magnetic FF might differ substantially. Note also that the new data about G_{Ep} in the SL region do not change essentially $|G_M|$ in the TL region, in comparison with the standard analysis (cases 1 and 2).

Fig. 2b shows the asymmetry for cases 3 and 4. Case 1 and case 2 give, respectively, $\mathcal{A} = 1$ and $\mathcal{A} = (\tau - 1)/(\tau + 1)$.

The predicted asymmetry is very sensitive to the different underlying assumptions.

The measurement of the differential cross section for the process $\bar{p} + p \rightarrow e^+ + e^-$ at a fixed value of s and for two different angles θ , allowing the separation of the two FFs, $|G_M|^2$ and $|G_E|^2$, is equivalent to the well known Rosenbluth separation for the elastic ep -scattering. However in TL, this procedure is simpler, as it requires to change only one kinematical variable, $\cos \theta$, whereas, in SL it is necessary to change simultaneously two kinematical variables: the energy of the initial electron and the electron scattering angle, fixing the momentum transfer square, q^2 .

The angular dependence of the cross section, Eq. (2), results directly from the assumption of one-photon exchange, where the spin of the photon is equal 1 and the electromagnetic hadron interaction satisfies the C -invariance. Therefore the measurement of the differential cross section at three angles would allow to test the presence of 2γ exchange also. The interference of C-odd amplitude of the one-photon exchange with the C-even amplitude of the two-photon exchange, will give rise to odd $\cos \theta$ -terms in the cross section:

$$\frac{d\sigma}{d(\cos \theta)}(\bar{p}p \rightarrow e^+e^-) = \sigma_0 \left[1 + a_1 \cos \theta + a_2 \cos^2 \theta + a_3 \cos^3 \theta + \dots \right], \quad (5)$$

where a_i , $i = 1, 2, 3..$, are s -dependent real coefficients.

In order to cancel the possible effects which can be induced by the two-photon contribution, one can suggest the following procedure:

- Consider the sum of the differential cross section at two angles, θ and $\pi - \theta$, i.e.:

$$\frac{d\sigma}{d\cos\theta}(\theta) + \frac{d\sigma}{d\cos\theta}(\pi - \theta) = 2\sigma_0 [1 + a_2 \cos^2 \theta \dots] \quad (6)$$

- Solve the following integral in which the $\cos\theta$ -odd terms disappear:

$$\int_{-\cos\theta_{max}}^{\cos\theta_{max}} \frac{d\sigma}{d\cos\theta}(\theta) d\cos\theta = 2\sigma_0 \cos\theta_{max} [1 + 3a_2 \cos^2 \theta_{max} + \dots] \quad (7)$$

On the other hand the following difference is very sensitive to the $1\gamma \times 2\gamma$ -interference contribution:

$$\frac{d\sigma}{d\cos\theta}(\theta) - \frac{d\sigma}{d\cos\theta}(\pi - \theta) = 2\sigma_0 [a_1 \cos\theta + a_3 \cos^3 \theta \dots] \quad (8)$$

The integration gives:

$$\int_0^{\cos\theta_{max}} \frac{d\sigma}{d\cos\theta}(\theta) d\cos\theta - \int_{-\cos\theta_{max}}^0 \frac{d\sigma}{d\cos\theta}(\theta) d\cos\theta = 2\sigma_0 \cos\theta_{max} \left[a_1 + \frac{1}{2} a_3 \cos^2 \theta_{max} + \dots \right] \quad (9)$$

A similar procedure can be suggested in the SL region, through the comparison of electron and positron scattering in the same kinematical conditions.

The relative role of the 2γ mechanism can increase at relatively large momentum transfer in SL and TL regions, for the same physical reasons, which are related to the steep decreasing of the hadronic electromagnetic FFs, as previously discussed in [25-28] and more recently in [29].

Let us summarize our conclusions about the properties of nucleon electromagnetic FF in the time-like region.

- The electric FF of the proton, which can be derived from the $\bar{p} + p \rightarrow e^+ + e^-$ data, in the hypothesis $|G_M| = |G_E|$ or $G_M^{(TL)}(s) = G_M^{(SL)}(q^2)$ ($s = -q^2$), strongly deviates from the measured values of $G_E^{(SL)}(q^2)$, and from asymptotic expectations.
- The measurement of the asymmetry \mathcal{A} of the angular dependence of the differential cross section for $\bar{p} + p \leftrightarrow e^+ + e^-$ is sensitive to the relative value of G_M and G_E .

- An extrapolation to high q^2 of the TL experimental data indicates that the Phr gmen-Lindel f theorem will be satisfied by the magnetic proton FF, only for $s(q^2) \geq 20$ (GeV/c)². This conclusion is nearly independent on different assumptions concerning $|G_E|$.
- The value of the mass parameter m_{nd}^2 , which characterizes the deviation of $G_M^{(TL)}$ from the dipole dependence, is comparable for the magnetic proton FF in TL region, for the electric proton FF in SL region and for the magnetic FF of the $N \rightarrow \Delta$ transition and may indicate that this parameter is related to the internal hadronic electromagnetic structure.
- The presence of a large relative phase of magnetic and electric proton FF in the TL region, if experimentally proven at relatively large momentum transfer, can be considered a strong indication that these FFs have a different asymptotic behavior.

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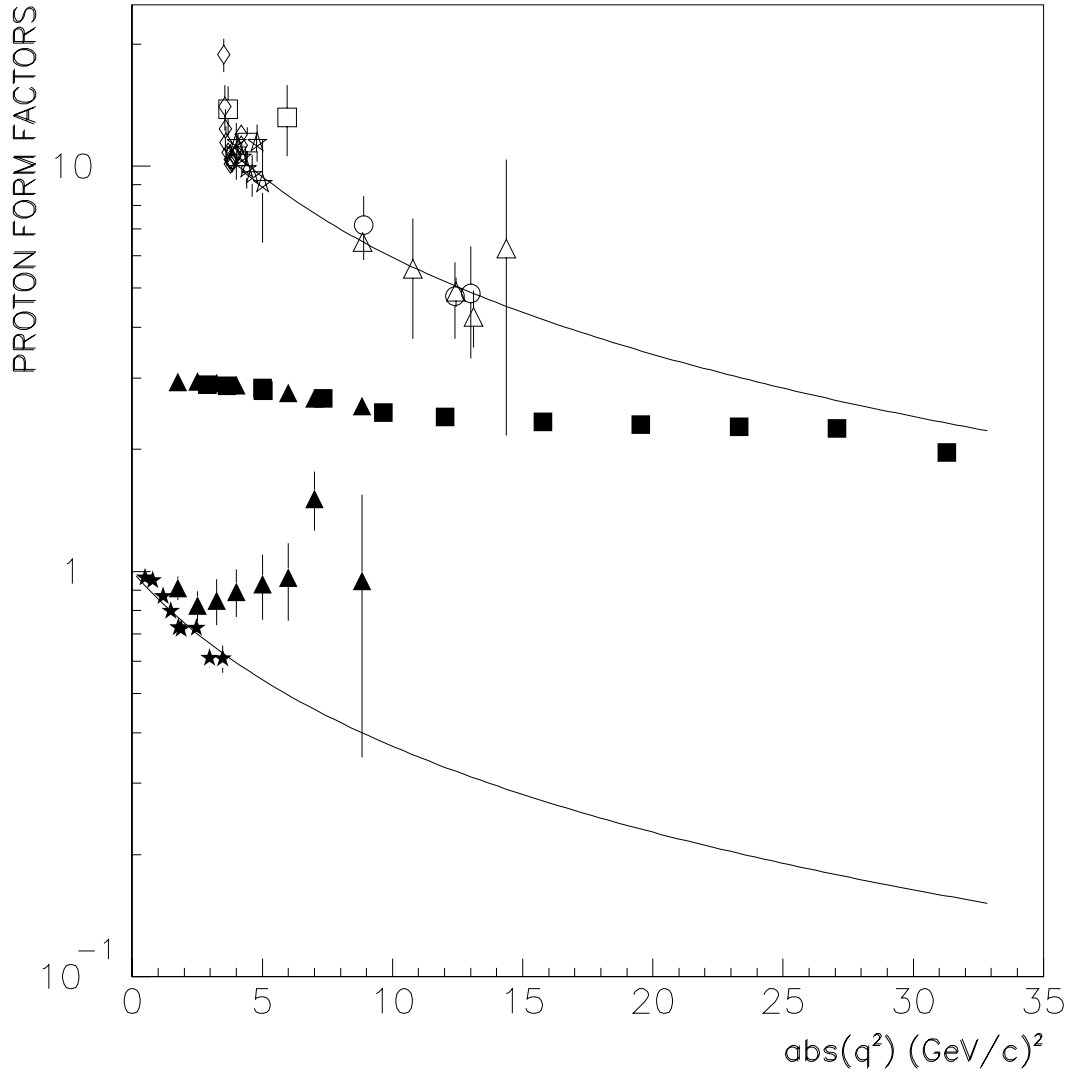


Figure 1: Existing data for electric and magnetic form factors in space like and time-like regions, scaled by dipole, as functions of the modulus of q^2 . Data in space-like region are taken from: [2] (stars), [18] (solid triangles), [19] (solid squares). Data in time-like region are taken from: [21] (open circles), [22] (open squares), [23] (open diamonds) [1] (open triangles), [24] (open stars).

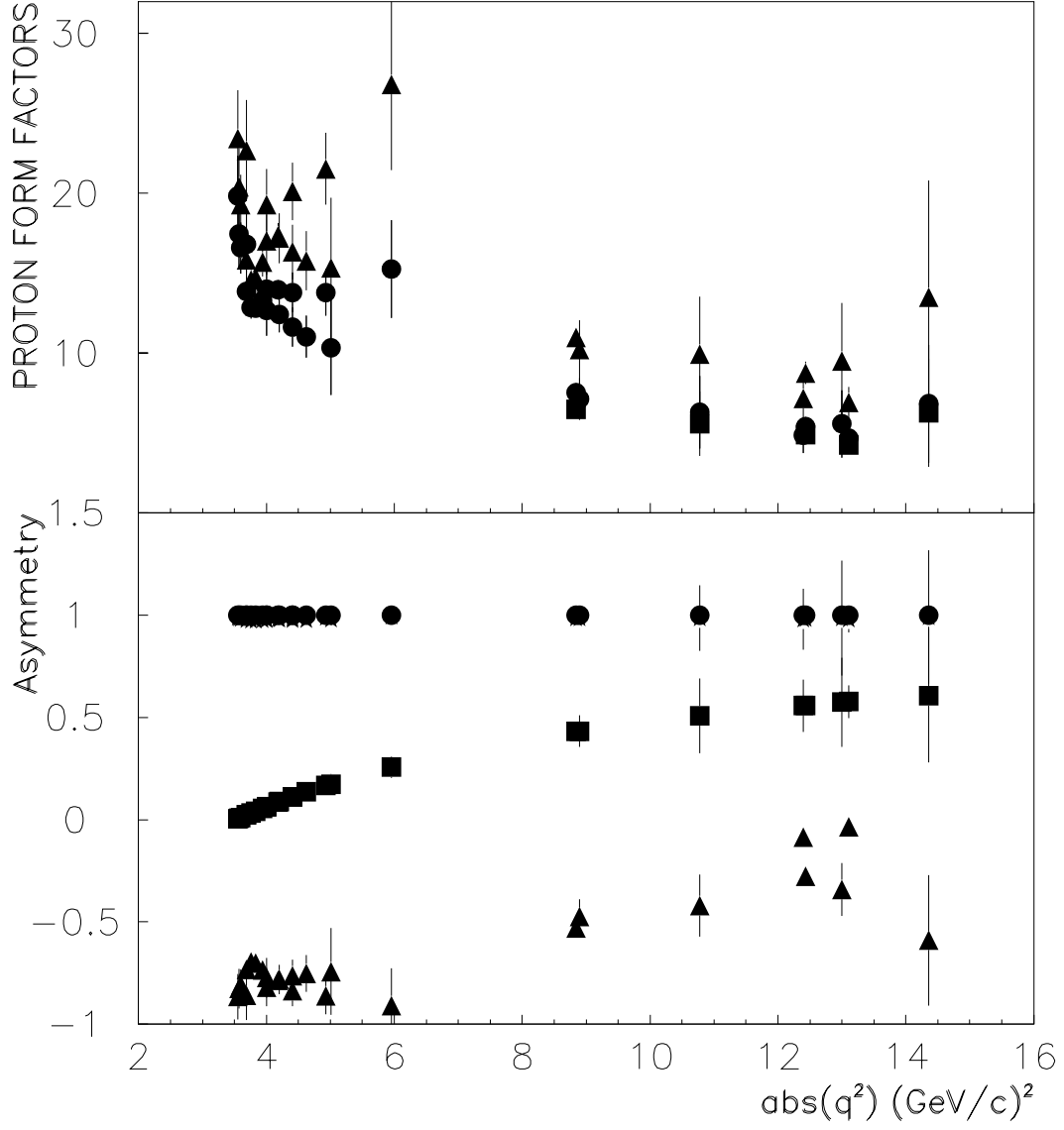


Figure 2: Nucleon form factors (top) and asymmetry (bottom) in TL region, deduced from the data following different assumptions (see text). case 1: $G_E = 0$ (circles); case 2: $G_E = G_M$ (squares); case 3: $G_M = \text{dipole}$ (triangles); case 4: G_E from [2] (stars).